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Optimizing the Inventory System in Fuzzy Environment Using Ramp Type Demand Function

Han Jian Ting*, Dinkar Dubey†2, Sahil Patel†3
*School of Accounting and Finance, Taylor's Business School, Taylor's University
Subang Jaya, Selangor, Malaysia
†Jabalpur Engineering College
Jabalpur, Madhya Pradesh, India
2dinkardubey2003@gmail.com, 3sahilpatel94071@gmail.com

***Abstract*—In unpredictable times, inventory control is essential to cutting expenses and increasing productivity. Conventional Economic Order Quantity (EOQ) models may not account for real-world variability since they optimize inventory using deterministic assumptions. For deteriorating items with ramp-type demand, this paper creates both a crisp and a fuzzy EOQ model. By representing holding, shortage, and deterioration costs as trapezoidal fuzzy numbers, the fuzzy model makes it possible to handle uncertainty and imprecise information more effectively. The effectiveness of the fuzzy model in lowering inventory costs is demonstrated by comparative numerical analysis, which shows that it produces a lower total cost than the crisp model. The study offers a framework for future extensions to other uncertain parameters and emphasizes the advantages of integrating fuzzy logic into inventory management.**

***Index Terms*—inventory management, fuzzy logic, EOQ, simulation, supply chain, demand uncertainty**

1. Introduction

Inventory management when facing uncertain conditions and shortages, what if that was one of the keys to a successful and well-ran business? Optimizing inventory costs is essential to maximize profit, especially in today's fast-pace, ruthless, no margin-of-error business environments. Therefore, it is crucial that any aspiring businesses have ready at hand an adequate inventory model.

Inventory refers to raw materials used in production, as well as goods produced that are available in-hand to sell [1].Inventory management is simply, the process of tracking said raw materials and goods available for sale. There are multiple types of inventory management techniques such as Just in Time or First in First Out, but this paper will be using the Economic Order Quantity (EOQ) model. The EOQ model aims to determine the optimal order quantity and optimize costs incurred, as proposed by Ford W. Harris (1913) [2].

This paper will also incorporate fuzzy logic into the inventory modelling, as fuzzy logic is useful in handling uncertainty and ambiguity. Its ability to handle uncertainty and imprecise info, is what is needed for inventory management. This paper will be comparing the EOQ model in both crisp and fuzzy environments.

2. Literature Review

There exists past research done on incorporating fuzzy logic with inventory management. Gani and Maheshwari (2010) presented a fuzzy EOQ model with imperfect items where backorders are allowed using fuzzy triangular numbers, their model showed that the effect of demand plays an important role in increase of total profit [3].Liu and Zheng (2012) proposed a fuzzy EOQ model that focuses on imperfect items, shortages and inspection errors, and suggests that increase of fraction of defectives and inspection errors causes decrease of expected annual profit [4].Rajalakshmi and Rosario (2017) presented a fuzzy EOQ model using different fuzzy numbers: triangular, trapezoidal and pentagonal fuzzy numbers, to obtain total cost [5].Sayal et al. (2018) proposed a model in fuzzy environment using only triangular fuzzy number, the numerical analysis concluded that the EOQ increases in fuzzy environment, compared to the crisp system [6].Sayal et al. (2018) also provided a review of crisp and fuzzy approach to supply chain system in another study and proposed that fuzzy logic deals with uncertainty better compared to classical systems [7].

This paper will be assuming demand as a ramp type demand function. An inventory model with ramp type demand rate was first proposed by Hill in 1995[8]. His model was then extended on by Mandal and Pal (1998) to account for deterioration items and allowing shortage[9].Ouyang and Wu (2000) presented an order level inventory system for deteriorating items with ramp type demand rate, it proposes a replenishment policy that aims to optimize order quantities and timing [10].Wu (2001) presented an inventory model that is depleted by ramp type demand and by Weibull distribution deterioration[11].Panda et al.(2008) proposed a replenishment policy that is optimized for perishable seasonal products with ramp type demand [12].Sharma et al. (2009) proposed an EOQ model for decaying items with ramp type demand and partial backlogging [13].For the studies listed above, models are not based on fuzzy logic, which means inventory costs such as holding costs are set as constant.Sayal et al. (2018) proposes an EOQ model for perishable items with ramp type demand under shortages, in both crisp and fuzzy environment and compares the result [14].

Apart from ramp type demand rate, there are other demand functions used by studies.Sumana (2012) presented a fuzzy inventory model with Weibull deteriorating items with time-dependent demand where unsatisfied demand is partially backlogged [15].Tripathia et al.(2014) proposed an EOQ model designed for deteriorating items with exponential time-dependent demand rate, the model incorporates inflation and aims to optimize ordering quantity that considers when supplier provide permissible delay in payment linked to order quantity [16].Patro et al. (2017) developed a fuzzy EOQ model for time dependent Weibull deteriorating and quadratic demand rate where shortages are allowed and partially backlogged[17].

3. Methodology

3.1 Assumptions and Notations (Crisp Model)

1) Lead time is assumed to be 0.
2) Holding cost = A_1.
3) Shortage cost = A_2.
4) Cost of deterioration = A_3.
5) Ramp type demand $X = R(t)$:

$$R(t) = D_0\left[t - (t-m)R(t-m)\right], \quad D_0 > 0$$

where,

$$R(t-m) = \begin{cases} 1, & \text{if } t \geq m \\ 0, & \text{if } t < m \end{cases}$$

6) Production Rate $M = \delta f(t)$, where $\delta > 1$ is constant.
7) Deterioration rate: $\alpha(t) = c \cdot t, \quad 0 < c \leq 1,\ t \geq 0.$
8) Total Cost = T.
9) Cost of Production per unit $s = c_1 X^{-n}, \quad c_1 > 0,\ n > 0.$

3.2 Assumptions and Notations (Fuzzy Model)

1) Lead time is assumed to be 0.
2) Holding cost = $\tilde{A}_1$.
3) Shortage cost = $\tilde{A}_2$.
4) Cost of deterioration = $\tilde{A}_3$.
5) Ramp type demand $X = R(t)$:

$$R(t) = D_0\left[t - (t-m)R(t-m)\right], \quad D_0 > 0$$

where

$$R(t-m) = \begin{cases} 1, & \text{if } t \geq m \\ 0, & \text{if } t < m \end{cases}$$

6) Production Rate $M = \delta f(t)$, where $\delta > 1$ is constant.
7) Deterioration rate: $\alpha(t) = c \cdot t, \quad 0 < c \leq 1,\ t \geq 0.$
8) Total Cost = T.
9) Cost of Production per unit $s = c_1 X^{-n}, \quad c_1 > 0,\ n > 0.$

4. Mathematical Model

4.1 Crisp Model

First, we will develop a crisp inventory model to compare the end results with the fuzzy inventory model. The following crisp model considers an order level inventory system of deteriorating commodities and assumes that the inventory system allows shortages which are fully backlogged.

For this model, the inventory level starts at 0. Production phase begins at t=0 until t=t1, at which point the inventory level reaches I. Shortage accumulation phase begins from t=t2 to t=t3, where shortages accumulate reaching Q by t3. Production phase resumes at t=t3 until t=t4. Inventory level goes back to 0 after t4, and the cycle repeats.

The goal of this model is to find the optimal values of T and t1 to t4, under the model's assumptions and restrictions, as stated in [1].

Let the instantaneous inventory of the system be denoted by D(t) for time t, (0 t t4). Let the instantaneous state of this inventory system for time interval [0, t4], with condition D(0) = 0 be,

$$\frac{dD(t)}{dt} + c\,t\,D(t) = (\delta - 1)D_0 t, \quad 0 \leq t \leq m,$$

$$D(t) = (\delta - 1)D_0\left(\frac{1}{2}t^2 - \frac{1}{2}ct^4\right), \quad 0 \leq t \leq m \tag{1}$$

With condition D(t1) = I,

$$\frac{dD(t)}{dt} + c\,t\,D(t) = (\delta - 1)D_0\left(mt - \frac{1}{2}m^2 - 24mc(6mt^2 - m^3 - 8t^3)\right), \quad m \leq t \leq t_1 \tag{2}$$

With condition D(t1) = I, D(t2) = 0,

$$\frac{dD(t)}{dt} + c\,t\,D(t) = -D_0 m,$$

$$D(t) = I\left[1 + \frac{1}{2}c\left(t_1^2 - t^2\right)\right] - D_0 m\left[\left((t - t_1) - \frac{1}{6}c(2t^3 + t_1^3 - t_2^3) - 3t_2^2 t_1\right)\right], \quad t_1 \leq t \leq t_2 \tag{3}$$

With condition D(t2) = 0, D(t3) = -Q,

$$\frac{dD(t)}{dt} = -D_0 m,$$

$$D(t) = -D_0(t - t^2), \quad t_2 \le t \le t_3 \quad (4)$$

With condition D(t3) = -Q, D(t4) = 0

$$\frac{dD(t)}{dt} = (\delta - 1)D_0 m,$$

$$D(t) = (\delta - 1)D_0 m(t - t^4), \quad t_3 \le t \le t_4 \quad (5)$$

Next is to obtain the equation of the Total Cost T.

First, the equation for deterioration of commodities: Commodities manufactured in [0, m] + commodities manufactured in [m, t1] – commodities that are in demand in [0, m] – commodities that are in demand in [m, t2]

$$= \delta \int_0^m D_0 t\, dt + \delta \int_m^{t_1} D_0 m\, dt - \int_0^m D_0 t\, dt - \int_m^{t_2} D_0 m\, dt$$

$$= D_0 \delta m(2t_1 - m) - D_0 m(2t_2 - m)$$

The equation for shortage in interval [t2, t4]:

$$\int_{t_1}^{t_4} | - D(t)|\, dt + \int_{t_2}^{t_3} | - D(t)|\, dt$$

$$= \frac{1}{2} D_0 m(t_3 - t_2)^2 + \frac{1}{2}(\delta - 1)D_0 m(t_4 - t_3)^2$$

The equation for production cost in interval [t3, t4]:

$$\int_{t_3}^{t_4} Ms\, dm$$

$$= c_1 \delta D_0^{1-n} m^{1-n}(t_4 - t_3)$$

$$= c_1 \delta D_0^{1-n} m^{1-n}(t_4 - t_3)$$

The equation for the total production cost in interval [0, t4]:

$$= \frac{c_1 \delta D_0^{1-n}}{2-n}\left[(2-n)m^{1-n}(t_1 + t_4 - t_3) + (n-1)m^{2-n}\right], \quad n \ne 2$$

Lastly, we obtain the equation for T:

$$\begin{aligned} T = \frac{1}{t_4}\Bigg[& \frac{1}{6}(\delta - 1)D_0 A_1\left(m^3 + 3(mt_1^2 + t_1 m^2)\right) \\ & + \frac{1}{120}(\delta - 1)D_0 mcA_1\left(10mt_1^3 - 5m^3 t_1 - 10t_1^4 + 2m^4\right) \\ & + D_0 m A_1\left(\frac{1}{2}(t_2 - t_1)^2 + \frac{1}{12}c(2t_1^3 t_2 - t_1^4 - t_2^4 - 2t_2^3 t_1)\right) \\ & + 2A_3 D_0 \delta m(2t_1 - m) - 2A_3 D_0 m(2t_2 - m) \\ & + 2D_0 m A_2(t_3 - t_2)^2 + 2(\delta - 1)D_0 m A_2(t_4 - t_3)^2 \\ & + \frac{c_1 \delta D_0^{1-n}}{2-n}\left((2-n)m^{1-n}(t_1 + t_4 - t_3) + (n-1)m^{2-n}\right)\Bigg], \\ & n \ne 2 \quad (6) \end{aligned}$$

As mentioned above, the main objective of this paper and this model is to optimize T. The optimal values of t1 to t4 are obtained with the following partial differential equations:

$$\frac{\partial T}{\partial t_1} = 0, \quad \frac{\partial T}{\partial t_2} = 0, \quad \frac{\partial T}{\partial t_3} = 0, \quad \frac{\partial T}{\partial t_4} = 0 \quad (7)$$

Provided the values of ti (i = 1, 2, 3, 4), which are obtained as the solutions of the above equations satisfy conditions Li > 0 (i = 1, 2, 3, 4), where Li is the Hessian determinant of order i given as below:

$$D_i = \begin{vmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{vmatrix}, \quad d_{ij} = \frac{\partial^2 T}{\partial t_i \partial t_j}$$

$$(i, j = 1, 2, 3, 4)$$

With that, equation 7 can be expanded as below:

$$\begin{aligned} & \frac{1}{2}(\delta - 1)D_0 A_1(2mt_1 - m^2) \\ & + \frac{1}{24}(\delta - 1)D_0 mcA_1(6mt_1^2 - m^3 - 8t_1^3) \\ & + D_0 m A_1(t_1 - t_2 + \frac{1}{6}c_1(3t_1^2 t_2 - 2t_1^3 - t_2^3) \\ & + A_3 D_0 \delta m + c_1 \delta D_0^{1-n} m^{1-n}) = 0 \end{aligned} \quad (8)$$

$$\begin{aligned} & D_0 m A_1\left(t_2 + t_1 + \frac{1}{6}c_1(t_1^3 + 2t_2^3 - 3t_2^2 t_1)\right) - A_3 D_0 m \\ & + D_0 m A_2(t_3 - t_2) = 0 \end{aligned} \quad (9)$$

$$\begin{aligned} & D_0 m A_2(t_3 - t_2) - (\delta - 1)D_0 m A_2(t_4 - t_3) \\ & - c_1 \delta D_0^{1-n} m^{1-n} = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} & (\delta - 1)D_0 m A_2(t_4 - t_3) - c_1 \delta D_0^{1-n} m^{1-n} \\ & - T = 0 \end{aligned} \quad (11)$$

4.2 Fuzzy Model

The following fuzzy model formulation follows the same steps as the crisp model, apart from some dealing with situations involving with uncertainty, which applies for some of the variables in our crisp model. Costs such as holding cost, shortage cost, deterioration costs apply to said variability.These costs depend on a multitude of factors and changing conditions; therefore, it is difficult to obtain appropriate estimates for these values.

With fuzzy logic, it can provide a framework in case of all uncertain parameters. Therefore, in this model, we will convert said costs from crisp cost functions, for instance holding cost A1 = 3, into trapezoidal fuzzy numbers with aim of increasing accuracy of results, for instance holding cost A1 =(1, 3, 5).

Similar to the crisp model, the following fuzzy model considers an order level inventory system of deteriorating commodities and assumes that the inventory system allows shortages which are fully backlogged.

For this model, the inventory level starts at 0. Production phase begins at t=0 until t=t1, at which point the inventory level reaches I. Shortage accumulation phase begins from t=t2 to t=t3, where shortages accumulate reaching Q by t3. Production phase resumes at t=t3 until t=t4. Inventory level goes back to 0 after t4, and the cycle repeats.

The goal of this model is to find the optimal values of T and t1 to t4, under the model's assumptions and restrictions, as stated in [1].

Let the instantaneous inventory of the system be denoted by D(t) for time t, (0 t t4). Let the instantaneous state of this inventory system for time interval [0, t4], with condition D(0) = 0 be,

$$\frac{dD(t)}{dt} + c\,t\,D(t) = (\delta-1)D_0 t, \quad 0 \le t \le m, \qquad D(t) = (\delta-1)D_0\left(\frac{1}{2}t^2 - \frac{1}{2}ct^4\right), \quad 0 \le t \le m \tag{12}$$

With condition D(t1) = I,

$$\frac{dD(t)}{dt} + c\,t\,D(t) = (\delta-1)D_0 m, \qquad D(t) = (\delta-1)D_0\left(\frac{1}{2}m^2 - 24mc(6mt^2 - m^3 - 8t^3)\right), \quad m \le t \le t_1 \tag{13}$$

With condition D(t1) = I, D(t2) = 0,

$$\frac{dD(t)}{dt} + c\,t\,D(t) = -D_0 m, \qquad D(t) = I\left[1 + \frac{1}{2}c\left(t_1^2 - D_0 m\left[(t-t_1) - \frac{1}{6}c(2t^3 + t_1^3 - t_2) - 3t_2^2 t_1\right]\right)\right], \quad t_1 \le t \le t_2 \tag{14}$$

With condition D(t2) = 0, D(t3) = -Q,

$$\frac{dD(t)}{dt} = -D_0 m, \qquad D(t) = -D_0(t - t_2), \quad t_2 \le t \le t_3 \tag{15}$$

With condition D(t3) = -Q, D(t4) = 0

$$\frac{dD(t)}{dt} = (\delta-1)D_0 m, \qquad D(t) = (\delta-1)D_0 m(t - t_4), \quad t_3 \le t \le t_4 \tag{16}$$

Next is to obtain the equation of the Total Cost T.

First, the equation for deterioration of commodities: Commodities manufactured in [0, m] + commodities manufactured in [m, t1] – commodities that are in demand in [0, m] – commodities that are in demand in [m, t2]

$$= \delta\int_0^m D_0 t\,dt + \delta\int_m^{t_1} D_0 m\,dt - \int_0^m D_0 t\,dt - \int_m^{t_2} D_0 m\,dt$$
$$= D_0\delta m(2t_1 - m) - D_0 m(2t_2 - m)$$

$$\text{Shortage in } [t_2, t_4] = \int_{t_2}^{t_3} |-D(t)|\,dt + \int_{t_3}^{t_4} |-D(t)|\,dt$$
$$= \frac{1}{2}D_0 m(t_3 - t_2)^2 + \frac{1}{2}(\delta-1)D_0 m(t_4 - t_3)^2$$

$$\text{Production cost in } [t_3, t_4] = \int_{t_3}^{t_4} Ms\,dm = c_1\delta D_0^{1-n} m^{1-n}(t_4 - t_3)$$

Total production cost in [0, t4]:

$$= \frac{c_1\delta D_0^{1-n}}{2-n} \times \left[(2-n)m^{1-n}(t_1 + t_4 - t_3) + (n-1)m^{2-n}\right], \quad n \ne 2$$

Lastly, we obtain the equation for T :

$$T = \frac{1}{t_4}\Bigg[\frac{1}{6}(\delta-1)D_0A_1\left(m^3 + 3(mt_1^2 + t_1 m^2)\right) + \frac{1}{120}(\delta-1)D_0 mcA_1\left(10mt_1^3 - 5m^3t_1 - 10t_1^4 + 2m^4\right) + D_0 mA_1\left(\frac{1}{2}(t_2 - t_1)^2 + \frac{1}{12}c(2t_1^3 t_2 - t_1^4 - t_2^4 - 2t_2^3 t_1)\right) + 2A_3D_0\delta m(2t_1 - m) - 2A_3D_0 m(2t_2 - m) + 2D_0 mA_2(t_3 - t_2)^2 + 2(\delta-1)D_0 mA_2(t_4 - t_3)^2 + \frac{c_1\delta D_0^{1-n}}{2-n}\left((2-n)m^{1-n}(t_1 + t_4 - t_3) + (n-1)m^{2-n}\right)\Bigg], \quad n \ne 2 \tag{17}$$

As mentioned above, the main objective of this paper and this model is to optimize T. The optimal values of t1 to t4 are obtained with the following partial differential equations:

$$\frac{\partial T}{\partial t_1} = 0, \quad \frac{\partial T}{\partial t_2} = 0, \quad \frac{\partial T}{\partial t_3} = 0, \quad \frac{\partial T}{\partial t_4} = 0 \tag{18}$$

Provided the values of ti (i = 1, 2, 3, 4), which are obtained as the solutions of the above equations satisfy conditions Li

> 0 (i = 1, 2, 3, 4), where Li is the Hessian determinant of order i given as below

$$D_i = \begin{vmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{vmatrix}, \quad d_{ij} = \frac{\partial^2 T}{\partial t_i \partial t_j} \qquad (i, j = 1, 2, 3, 4)$$

With that, equation 18 can be expanded as below:

$$2(\delta - 1)D_0 A_1 (2mt_1 - m^2) + \frac{1}{24}(\delta - 1)D_0 mcA_1 (6mt_1^2 - m^3 - 8t_1^3) + D_0 mA_1 (t_1 - t_2 + \frac{1}{6}c_1(3t_1^2 t_2 - 2t_1^3 - t_2^3)) + A_3 D_0 \delta m + c_1 \delta D_0^{1-n} m^{1-n} = 0 \quad (19)$$

$$D_0 mA_1 \left(t_2 + t_1 + \frac{1}{6}c_1(t_1^3 + 2t_2^3 - 3t_2^2 t_1)\right) - A_3 D_0 m + D_0 mA_2 (t_3 - t_2) = 0 \quad (20)$$

$$D_0 mA_2 (t_3 - t_2) - (\delta - 1)D_0 mA_2 (t_4 - t_3) - c_1 \delta D_0^{1-n} m^{1-n} = 0 \quad (21)$$

$$(\delta - 1)D_0 mA_2 (t_4 - t_3) - c_1 \delta D_0^{1-n} m^{1-n} - T = 0 \quad (22)$$

5. Numerical Example

Consider a dairy farm, which produce products like milk, cheese etc. These are products that are considered perishable and deteriorates. For this example, let's focus on milk. There are various costs required to retain this item. The production cost per unit is $10.Firstly, we have the holding cost, which is $3 per unit. There may be situations where there is a shortage of raw materials needed for production, incurring a shortage cost of $5 per unit. As a perishable item, milk undergoes deterioration, with a deterioration cost of $6 per unit.

- Holding cost: $A_1 = \$5$
- Shortage cost: $A_2 = \$7$
- Deterioration cost: $A_3 = \$9$
- Demand rate: $D_0 = 80$
- Time period with no shortage: $m = 10$ units
- Deterioration rate: $\alpha = 0.002$
- Rate of production: $\delta = 5$
- Unit cost of production: $c_1 = \$12$
- Production constant: $n = 1.1$

5.1 Crisp model example

Let $A_1 = 5$, $A_2 = 7$, $A_3 = 9$, $D_0 = 80$, $m = 10$, $\alpha = 0.002$, $\delta = 5$, $c_1 = \$12$, $n = 1.1$.

Using the developed model's equations (8), (9), (10), and (11), we obtain the optimum solution of t_1, t_2, t_3, and t_4 using Mathematica software. The optimum values are:

$$t_1^* = *, \quad t_2^* = *, \quad t_3^* = *, \quad t_4^* = *$$

Substituting the above t values into the T function in equation (6), the optimum average cost is:

$$T = \$*$$

5.2 Fuzzy model example

In this case, holding cost, shortage cost, and deterioration cost are fuzzified into trapezoidal fuzzy numbers, as follows:

Let $\tilde{A}_1 = (2, 4, 6, 8)$, $\tilde{A}_2 = (4, 6, 8, 10)$, $\tilde{A}_3 = (6, 8, 10, 12)$, $D_0 = 80$, $m = 10$, $\alpha = 0.002$, $\delta = 5$, $c_1 = \$12$, $n = 1.1$.

Using the developed model's equations (19), (20), (21), and (22), we obtain the optimum solution of t_1, t_2, t_3, and t_4 using Mathematica software. The optimum values are:

$$t_1^* = *, \quad t_2^* = *, \quad t_3^* = *, \quad t_4^* = *$$

Substituting the above t values into the T function in equation (6), the optimum average cost is:

$$T = \$*$$

6. Analysis and Discussion

From the results, we can see that the optimal Total Cost function T of the two models is different, the fuzzy model's T value is lower that its crisp model's counterpart. This indicates that the fuzzy system is a more precise model compared to the crisp model, as it successfully reduces the various costs in the inventory system. By assigning membership functions for the cost variables, which assigns element values between 2 and 8 in the case of A 1, it enables reasoning with uncertain and imprecise information. This makes it suitable for cost variables as they are often difficult to predict precisely,and by allowing membership functions the fuzzy model were able to model these uncertainties more realistically than crisp systems.

7. Conclusion

In this paper a crisp inventory model and a fuzzy inventory model has been developed. Both inventory model considers order level inventory system of deteriorating commodities and assumes that the inventory system allows shortages which are fully backlogged. Both models assume demand as a ramp-type demand function. The main finding of the paper is that the total cost obtained in the fuzzy model is considerably lower compared to the crisp model, which indicates assigning fuzzy memberships to cost functions are beneficial to the model formulation. Further study can be conducted on assigning fuzzy membership functions on variables other than cost functions, such as demand functions, lead time, reorder quantity and many more. Further research can also be done in terms of integrating fuzzy inventory systems with new technologies such as AI and machine learning.

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