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Comparative Analysis of Crisp and Fuzzy Inventory Management Systems

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ABSTRACT

Traditional Crisp (deterministic) inventory models and Fuzzy inventory management systems are analyzed and contrasted in this paper. The study assesses total cost, robustness to demand and lead-time variability, and the effect on profitability using a Mamdani-type fuzzy inference system (FIS) implemented in MATLAB and EOQ-based crisp simulation. Differences in total cost over 100 simulated periods are evaluated using multiple linear regression (MLR) and a statistical hypothesis test (t-test). The findings show that in the tested scenarios, the fuzzy approach provides better stability under uncertainty and lowers overall cost.

Index Terms:— *inventory management, fuzzy logic, EOQ, simulation, supply chain, demand uncertainty*

I. INTRODUCTION

1.1 Research Background

A good inventory system allows a business to minimise unnecessary holding costs, and maintaining appropriate stock levels prevents costs associated with sold-out goods and customer dissatisfaction, or over-ordering and excess stock. (Fernandez, M., 2024) Effective inventory management also provides valuable data and insights that can be used for future planning and decision making, and achieving the right balance through inventory control can reduce holding costs and improve profits. The main objective of this project is to analyse and compare inventory systems in crisp and fuzzy environments, with a focus on generating higher profits and reducing the total cost of inventory systems. By exploring both crisp and fuzzy systems, the project aims to find more optimal methods of inventory management.

The traditional crisp inventory system, also known as the deterministic inventory model is one that is based on the premise that all parameters and variables associated with inventory are known, and that there is no uncertainty in inventory demand and replenishment. These systems use precise fixed values to manage inventory levels and are designed to optimise order quantities and minimise total costs. The most common deterministic models used in inventory systems include the Economic Ordering Quantity (EOQ) model, ABC analysis, and inventory turnover ratio. (Melanie, 2019). However, these systems can struggle to cope with real-world uncertainty and variability, which can limit their effectiveness in dynamic and unpredictable environments. Crisp methods are beneficial due to their simplicity and clarity, but enhancements or alternatives are needed to better deal with uncertainty and improve overall efficiency, so this project is in order to apply fuzzification and de-fuzzification techniques to convert existing crisp systems to fuzzy systems to better improve the efficiency of inventory systems.

Fuzzy inventory management systems use fuzzy logic to deal with uncertainty and imprecision in inventory parameters, making them more adaptable to real-world changes. Fuzzy inventory management systems provide a powerful alternative to traditional crisp systems by effectively handling uncertainty and imprecision. Key concepts such as fuzzy set theory, fuzzification, fuzzy logic systems and defuzzification enable more powerful and adaptable inventory management strategies. Different function curves are calculated through complex mathematical logic to achieve fuzzification. The advantages of making it flexible, improving decision making and cost optimisation make it an invaluable tool for modern inventory management.

1.2 Problem Statement

In a crisp inventory management system, stock levels are managed using precise and fixed values, often assuming that parameters such as demand, lead times and costs are certain. However, in the real world, these parameters are prone to change and uncertainty, which can lead to inefficient inventory management processes. For instance, demand for winter gloves may rise if the winter turns out to be colder than expected. (ERP, 2024).Customer reorders will replenish inventory after the company has sold the product to the customer and determined that it needs to be restocked. But staff members who place repeat orders for commodities need to account for lead times—the interval of time between placing an order and the items arriving at the warehouse to be distributed. Lead times may lengthen and product availability may become problematic if the reordering procedure is delayed.

Crisp inventory systems are prone to inaccurate demand forecasting, which can lead to overstocking or out-of-stocking, and overstocking crises create additional tied-up capital for the industry, incurring higher storage costs and increasing the risk of obsolescence, especially for perishable or technologically advanced products, while understocking can lead to missed sales opportunities, customer dissatisfaction, and a potential loss of market share, which for the industry is a huge loss for the industry. In reality, retail chains may forecast demand based on last year's sales figures. However, if there is an unexpected surge in demand due to a new trend or seasonal change, the shop may be faced with out-of-stocks or over-stocks, resulting in lost sales or higher costs.

The crisp system assumes that lead times are fixed, but actual lead times may vary due to supplier reliability, shipping issues, or production delays. During Epidemic 2023, the automotive industry was alerted to the sudden stoppage of production at 14 Toyota assembly plants. A failure in the parts order management system prevented production from being re-procured. Toyota is known for its Just-in-Time (JiT) principles, which originated with the Toyota Production System (TPS) initiated by Kiichiro Toyoda. the JIT system means that production and shipping schedules are meticulously synchronised. This involves ordering materials to arrive on time when they are needed for manufacturing and shipping finished goods just in time to meet customer demand, thus effectively reducing warehousing costs. However, the success of this just-in-time approach depends on the timely delivery of all goods, parts or materials, which has not been the case since the outbreak. The global supply chain is out of sync, causing significant disruption, a classic case of a crisp inventory system. (To, S., 2023).

1.3 Research Questions

- How do fuzzy inventory management systems compare to crisp inventory systems in terms of cost efficiency and profitability?
- Can fuzzy systems better manage variability in demand and lead time, leading to more optimal inventory levels?
- What are the practical implications of implementing a fuzzy inventory

management system in a real-world organizational context?

- How do fuzzy inventory management systems impact customer satisfaction compared to traditional systems?

1.4 Research Objective

Develop and Compare Models Develop both crisp and fuzzy inventory management models and compare their performance under various scenarios. Evaluate Cost and Profit Assess the impact of each system on total inventory costs and profitability. Analyse Flexibility and Robustness Analyse the flexibility and robustness of fuzzy systems in managing inventory under uncertain conditions. Provide Recommendations Offer practical recommendations for organizations considering the adoption of fuzzy inventory management systems. Assess Impact on Customer Satisfaction Evaluate the effect of fuzzy inventory management systems on customer satisfaction levels.

1.5 Significance of Study

The significance of this study lies in its potential to provide valuable insights and practical solutions for improving inventory management practices in various industries. By comparing crisp inventory management systems with fuzzy inventory management systems, this research aims to highlight the advantages of incorporating fuzzy logic in handling uncertainty and variability in inventory parameters. The outcomes of this study could have far-reaching implications for businesses seeking to optimize their inventory processes, reduce costs, and enhance overall efficiency and customer satisfaction. Enhanced Decision Making Under Uncertainty Contribution: The study demonstrates how fuzzy inventory management systems can provide more accurate and adaptable decision -making frameworks under conditions of uncertainty and variability. Impact: This can help businesses better navigate the complexities of real-world demand and supply fluctuations, leading to more optimal inventory levels. Cost Efficiency and Profitability Contribution: By evaluating the cost efficiency and profitability of fuzzy systems compared to crisp systems, the study identifies potential cost savings and profit enhancement opportunities. Impact: Businesses can adopt fuzzy inventory management systems to achieve a better balance between ordering and holding costs, ultimately leading to lower total inventory costs and higher profitability. Improved Customer Satisfaction Contribution: The study examines the impact of fuzzy inventory management systems on customer satisfaction, particularly in terms of product availability and reduced stockouts. Impact: Higher customer satisfaction can result in increased customer loyalty, repeat business, and a stronger competitive position in the market.

Practical Recommendations Contribution: The study provides practical guidelines and best practices for implementing fuzzy inventory management systems in real-world organizational contexts. Impact: Businesses can leverage these recommendations to smoothly transition from crisp to fuzzy inventory management, minimizing potential challenges and maximizing benefits.

Inventory management is central to minimizing holding costs while avoiding stockouts that reduce customer satisfaction. Deterministic (crisp) models — such as Economic

Order Quantity (EOQ) — assume known, fixed parameters, but real-world conditions often involve uncertainty in demand and lead time. Fuzzy inventory systems apply fuzzy set theory, fuzzification and defuzzification to manage imprecision and variability, allowing inventory policies to adapt when parameters are uncertain. This study aims to develop crisp and fuzzy models, simulate them with realistic parameter variations (e.g., smartphone sales data from 2007–2023), and compare their performance primarily in terms of total cost and profitability.

II. LITERATURE REVIEW

2.1 Inventory Routing Problem in a Crisp (Deterministic) Environment

The Inventory Routing Problem (IRP) is one of the challenging optimisation problems in logistics and supply chain management. It focuses on the optimal integration of inventory control and vehicle routing operations in a supply network. The main objective is to determine the optimal distribution strategy, including a set of vehicle routes, required delivery quantities and delivery times, in order to minimise total inventory holding and transportation costs. The implementation of policies such as vendor-managed inventory (VMI) has been shown to significantly improve the overall performance of the supply network. (Radzuan, K., Rahim, M. K. I. A., Anuar, H. S., Nawi, M. N. M., Osman, W. N., 2015). Previous research has focussed on single-warehouse, multi-retailer vendor-managed inventory (SWMR-VMI), where all retailers face deterministic demand rates. However, in real-world problems, demand rates are usually not constant. Therefore, this paper will describe some publications on the Deterministic Inventory Routing Problem (DIRP).

Table 1 references the IRP options. For the first criterion, the time can be a limited or unlimited horizontal planning period. Retailer demand is the main issue in IRP. It consists of deterministic, stochastic and dynamic demand. In logistics, the number of suppliers and retailers may change, which may require one-to-one, one-to-many, and sometimes many-to-many relationships. Also, fleet composition and size can be homogeneous or heterogeneous. Homogeneous means that vehicles use a fixed capacity to deliver inventory on a route. Heterogeneous means that the distribution use and the number of available vehicles may be fixed to one, fixed to multiple, or unconstrained.

Figure 1 shows the route a vehicle should take to deliver an order to each retailer, where 0 represents a warehouse. The figure shows that the vehicle needs to decide which route is most conducive to reducing transport and inventory costs. In this case, it can be observed that the vehicle should take two routes to distribute the orders. For route A, the vehicle should make A = (0-1-4-3-0) and for route B = (0-2-5-0).

This is how distribution planning works to keep operations running smoothly.

Figure 2 shows a smaller vehicle capacity than the 30 tonnes shown in Figure 1. The vehicle is required to service five retailers but carries insufficient stock. In this case, the route must be extended to three routes to fulfil the requirement. The vehicle must take Route A = (0-1-4-0), Route B = (0-2-5-0) and Route C = (0-3-0). As can be seen from the above examples, the vehicle storage capacity has a significant impact.

Table 1: Variant Classification of IRP

Crite-ria	**Possible Options**		
Time	Finite	Infinite	Dynam-ic
De-mand	Determin-istic	Sto-chastic	Many-to -many
Struc-ture	One-to-one	One-to-many	
Rout-ing	Direct	Multi-ple	Continu-ous
Inven-tory Policy	Lost sales	Back-order	Non-negative
Fleet Com-position	Homoge-neous	Hetero-geneous	
Fleet Size	Single	Multi-ple	Uncon-strained

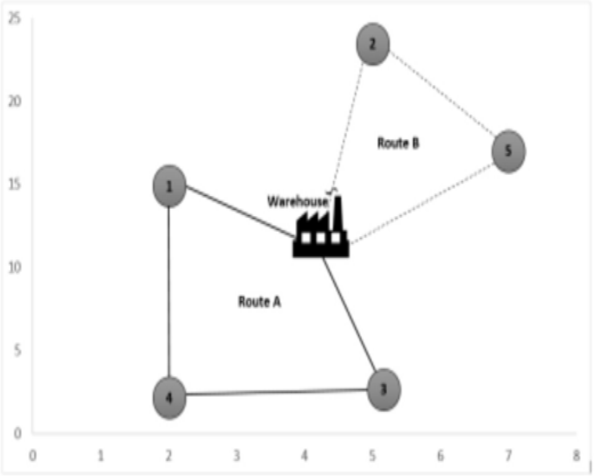

Figure 1: Distribution planning routes with vehicle capacity 50 tons

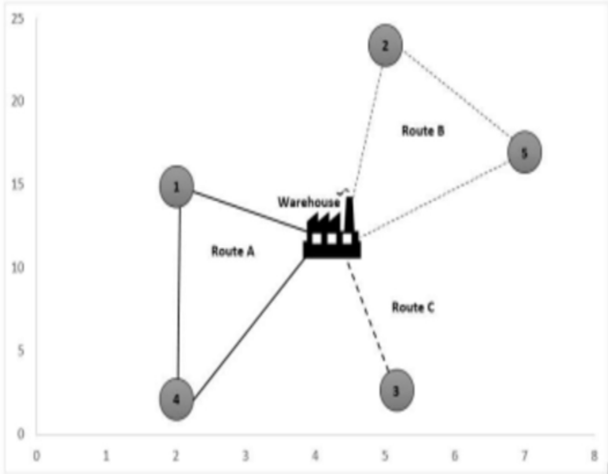

Figure 2: Distribution planning routes with vehicle capacity 30 tons

Figures 1 and 2 show that 50 tonnes and 30 tonnes of capacity are used to service each retailer respectively. In this case, not only does it show that vehicle capacity is important, but also that the most appropriate vehicle needs to be selected to reduce overall costs. For example, the total average cost for 50 tonnes is RM200 and the total amount for 30 tonnes is RM150. Therefore, it can be assumed that the smaller the capacity, the fewer retailers will be served and the more routes will be required. (Rahim, M.K.I.A. and Harahap, A.Z.M.K., 2017).

This paper investigates the Deterministic Inventory Routing Problem (DIRP) in which individual warehouses distribute individual products to retailers focusing on a fixed demand rate utilising a homogeneous fleet of vehicles for a given finite time horizon. The objective is to determine the desired quantity to be shipped to the retailers, the delivery time and to design the vehicle delivery routes in order to minimise the total distribution and inventory costs while ensuring that each retailer achieves a certain level of service quality at each time horizon of the planning horizon. This study gives us a clear view of the importance as well as limitations of route planning under deterministic inventory, which gives contrast for comparison fuzzy.

2.2 An Inventory Control System Using Fuzzy Logic

The study is a fuzzy logic-based inventory control model that considers a periodic review model for inventory control with variable order quantities. The model considers the dynamics of the production inventory system from the control theory point of view. The control module combines fuzzy logic with proportional-integral-derivative (PID) control algorithm. It simulates a decision support system to maintain finished goods inventory at the required level in case of changes in demand. The effectiveness of the proposed control model is illustrated with real data from a typical packaging organisation in the Sultanate of Oman.

Inventory control in production and operational systems is important for better management and utilisation of resources. This project proposes inventory ordering models from the perspective of control system theory, which take into account the non-linear nature of ordering rules. In these models, the current inventory level is compared with a pre-specified set value, and the comparison can be done periodically (at specified intervals) or continuously. Whenever the stock reaches the set value (trigger level), the order is triggered in the form of an impulse sequence. (D.R. Towill, S.S. Yoon, 1982).

The process is illustrated in two sets of simulation results using a packaging organisation in the Sultanate of Oman. In the first set, the annual average monthly production and sales data for a 3-year operating period (1995-1997) was used as the nominal capacity score. In the second set, actual monthly sales and production data for a 5-year period (1995-1999) were used as the fraction of rated capacity. For both sets of data, simulation models were built and then compared, and the results showed that inventories were reasonably well maintained at the desired levels despite changes in demand. The model can be used as a decision support system to monitor the inventory situation taking into account supply and demand scenarios.(B. Samanta, 2001).

2.3 Fully fuzzy inventory model with price-dependent demand and time varying holding cost under fuzzy decision variables

In this paper, an EOQ inventory model with price-related demand and time-varying holding costs in a fuzzy environment is considered using trapezoidal fuzzy numbers. A fully fuzzy inventory model is developed in which inputparameters and decision variables are fuzzified. For this fuzzy model, a defuzzified expected value approach is used to find an estimate of the profit function in a fuzzy sense. In addition, a rigorous method is constructed to test the optimal solution of the fully fuzzy inventory model. After defuzzification of the profit function, the proposed algorithm is used to determine the optimal strategy for the developed model.

Price-Dependent Demand Models: These models consider the demand for an item as a function of its price. Lee and Dye developed an inventory model for deteriorating items with stock-dependent demand and controllable deterioration rates. This model addresses practical challenges by adjusting to the dynamic nature of inventory demand and holding costs . (Lee, Y.-P., Dye, C.-Y., 2012). Fuzzy EOQ Models: Fuzzy set theory, introduced by Zadeh, has been extensively applied to inventory management to handle uncertainties in demand and supply parameters. (Zadeh, L.A., 1996). Mandal and Maiti developed a non-linear fuzzy model for a multi-item EOQ system, considering imprecise storage space and production run constraints . (Mondal, S. and Maiti, M., 2003).

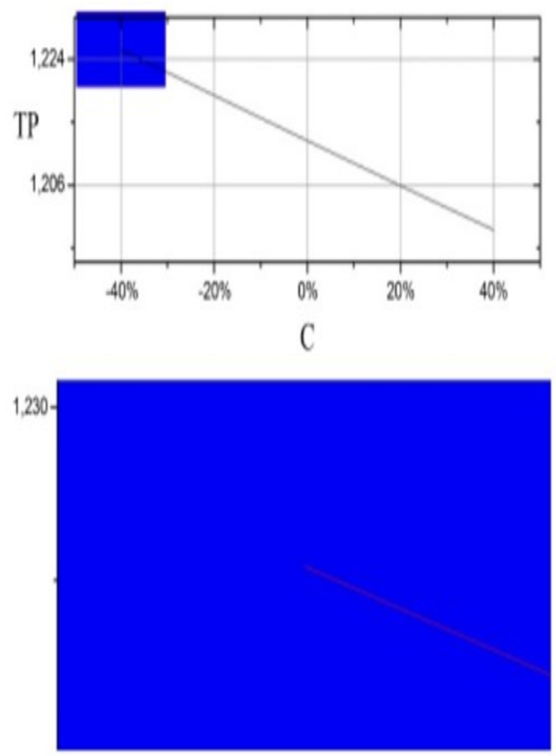

Figure 3: c̃ vs. max TP

We discuss here the fuzzy inventory model for goods with quantity discounts and no shortages. By fuzzifying the input parameters (P, Q, K, h, c, g, a and b) and decision variables (P and Q). Here, we assume that each input parameter is a non-negative trapezoidal fuzzy number consisting of nine components: Selling price: P~ = (P1, P2, P3, P4), order size: Q~ = (Q1, Q2, Q3, Q4), ordering cost: K~ = (K1, K2, K3, K4), cycle time: T~ = (T1, T2, T3, T4), unit purchasing cost: ~c = (c1, c2, c3, c4), time varying holding cost: h~ = (h1, h2, h3, h4), constant demand rate coefficient: a~ = (a1, a2, a3, a4), price-dependent demand rate coefficient: b~ = (b1, b2, b3, b4), constant holding cost coefficient: ~g = (g1, g2, g3, g4).

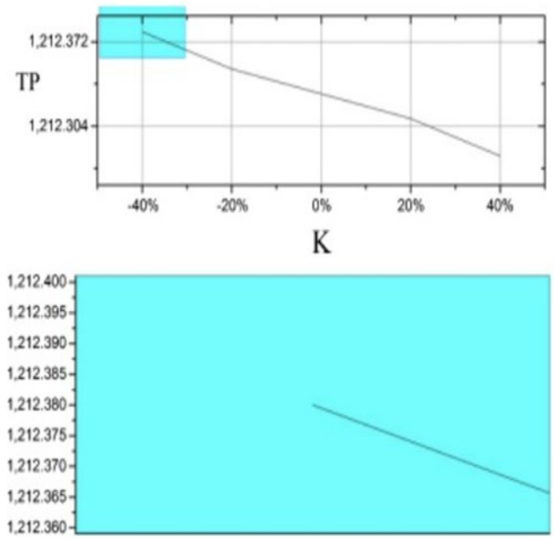

Figure 4: $\tilde{K}$ vs. min TC

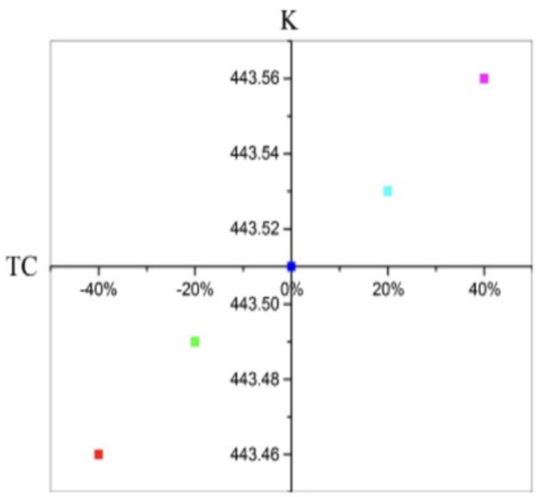

Figure 5: $\tilde{K}$ vs. max TP

After a complex mathematical transport, a systematic sensitivity analysis of the above example was carried out in order to assess the relative impact of different input parameters on the solution properties. The pic values of each given fuzzy parameter (~a, c, ~K~ and b~) were changed one at a time in relative steps of 20 % (-20%, -40%, +20 % and +40 %) and the impact on the best solution was recorded. Since four new values are considered for each of the four parameters, the sensitivity analysis requires the solution of 16 alternate problems. After calculating the sensitivity the data is displayed in the image table below.

Obviously, the unit purchase cost (~c) and the ordering cost (K~) are the most powerful factors for the profit function (TP) and the values of the variables Q, ~P, ~T~ and the total cost function (TC). This implies that in order to maximise prof-itability, firms should worry more about increasing demand than decreasing costs.

This paper presents a fully fuzzy inventory model with variable demand, variable holding cost and variable purchasing cost. In this model, the input parameters are represented as fuzzy numbers and the decision variables are considered as fuzzy numbers. The fully fuzzy inventory model with trapezoidal fuzzy numbers is solved using Lagrangian optimisation method. (Garai, T., Chakraborty, D. and Roy, T.K., 2019). The benefits of fuzzification for the industry and how it can be implemented to help companies achieve cost reduction can be better explained through this study.

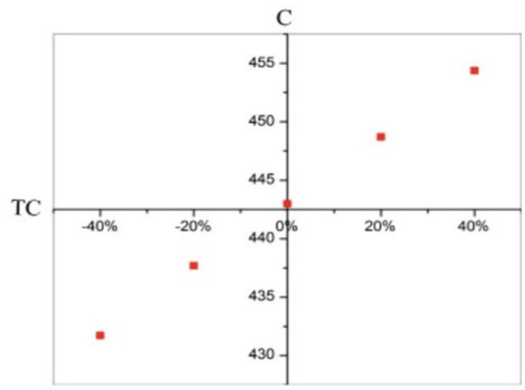

Figure. 6: $\tilde{c}$ vs. min TC

2.4 Developing a Fuzzy Logic-Based Carbon Emission Cost- Incorporated Inventory Model with Memory Effects

This paper explores the challenge of integrating carbon emission costs into inventory management to address global warming. Traditional inventory models lack consideration of memory effects and carbon emission costs. This study introduces a fuzzy logic-based inventory model that uses fractional order calculus to integrate memory effects and carbon emission costs.

The proposed model treats uncertain parameters such as ordering cost, spoilage rate and demand rate as triangular fuzzy numbers. Fractional order calculus is used to capture the memory effect, distinguishing between strong and weak memory effects through fractional order derivatives and integrals. The model aims to determine the optimal average cost and ordering interval using solution and sensitivity analysis. The memory effect significantly affects the performance and cost of the inventory system. Profitability is higher under strong memory effect as compared to weak memory effect. The signed distance method (SDM) produces higher profits than the graded mean integration method (GMIM), especially with the strong memory effect.

In summary, incorporating memory effects and carbon costs into inventory management can lead to higher profits and more sustainable practices. This study suggests that a strong memory effect improves profitability and that the SDM approach is more effective than GMIM. Future enhancements may include the impact of advertising on demand and backlog rates. This study provides a comprehensive understanding of how memory effects and carbon costs impact inventory systems, paving the way for more efficient and environmentally friendly inventory management practices. (Pakhira, R,2024) .

2.5 Integrated Fuzzy Inventory Model with Minimal Repair

This study proposes an integrated inventory model under fuzzy demand conditions that combines preventive and minimum repair maintenance strategies. The goal is to enhance supply chain management by developing robust inventory strategies that reflect realistic conditions. Numerical examples are provided to illustrate the application of the model.

Integrated Inventory Model: Goyal's initial model and subsequent enhancements consider coordinated efforts between suppliers and buyers to optimize inventory cycles and production quantities. Fuzzy Demand Model: Previous models addressed fuzzy demand and productivity issues to estimate costs and logistics in the supply chain.

Costs Considered:

- Cost of Preventive Maintenance (Cpm)
- Minimum Maintenance Cost (Cm)
- Cost of Inventory Holding (Ch)
- Cost of Ordering (Co)
- Cost of Purchasing (Cp)
- Setup Costs (Cs)
- Production Cost (Cv) Model Assumptions:
- Costs (setup, ordering, holding) are known constants.
- Initial production starts in a controlled state and produces a perfect product.
- Preventive maintenance is performed after production runs.
- Two types of preventive maintenance:
- Type I (Imperfect PM): the system maintains the same failure rate.
- Type II (Perfect PM): the system is restored to a brand new condition.
- Minimal maintenance allows the system to return to operation after a failure.
- Repair time is negligible and imperfections are fixed immediately.
- The inventory cycle has an unlimited time range.

The model considers fuzzy demand (triangular fuzzy number), preventive maintenance, and minimum repairs to compute the Joint Total Expected Annual Cost (JTEC).JTEC is a function of the inventory lead time (T) and the number of delivery lots (m). JTEC is a function of the inventory lead time (T) and the number of delivery lots (m). The model uses the signed distance method to convert fuzzy numbers to clear values, thus simplifying the calculation of total cost. It is shown that considering fuzzy demand and combining it with maintenance strategies can significantly affect inventory management efficiency. The proposed model provides flexibility and is more in line with real-world conditions, and it is recommended that other fuzzy factors be further explored in future research to enhance supply chain applications. (Yang, M.-F., Ko, M.-D. and Kuan, Y.-A, 2022)

3. *Methodology*

3.1 *Theoretical Framework*

This study defines the inventory differences used to analyze the research data to compare the crisp and fuzzy systems. In this study Matlab is used to simulate the crisp and fuzzy models with different fixed parameters such as demand rate, lead time, ordering cost, holding cost and also realistic data will be added to calculate the definition of the parameters. As well as simulation using fuzzy logic system which uses fuzzy logic to incorporate uncertainty and variability into the inventory management process and compare it with rigid EOQ model to see the difference in cost and profitability. Confirm the comparative analysis of inventory systems in crisp and fuzzy environments.

The hypothesis of this research is as follows:

- **Hypothesis(H1):** Implementing a fuzzy inventory management system will result in lower total costs compared to a crisp inventory management system.
- **Null Hypothesis (H0):** There is no significant difference in the total costs between the fuzzy inventory management system and the crisp inventory management system.

3.2 *Data Collection*

3.2.1 *Data Sources*

The source of data for this study is the calculation of demand parameters based on the number of smartphones sold to endusers globally from 2007 to 2023 as provided by the Statista website. Parameters are created through Matlab's model simulation program and a crisp inventory system is simulated for comparison with a fuzzy inventory system. Economic Order Quantity (EOQ) is used as the base model for the crisp inventory, the affiliation functions of the fuzzy parameters are defined, and fuzzification and defuzzification techniques are used. This study only considers inventory comparisons under a controlled model with set parameter ranges, which is observed by simulating actual variables and situations that may differ from the actual inventory environment.

3.2.2 *Variables*

3.2.2.1 *Independent Variables*

Demand Rate:

- The rate at which inventory is consumed or required over a specific period.
- It serves as an input to both the crisp and fuzzy inventory management systems. In the fuzzy model, it is a key factor that influences the order quantity decisions.

Lead Time:

- The time delay between placing an order and receiving the inventory.
- A critical input variable for both the crisp and fuzzy models. It affects how much inventory needs to be ordered to prevent stockouts.

3.2.2.2 *Dependent Variable Total Cost:*

- The sum of all costs associated with inventory management, including ordering costs, holding costs
- The primary output or outcome of the inventory management system
Total Fuzzy Cost C (T) . The research aims to minimize this variable to improve inventory efficiency.

3.3 *Assumptions*

The following assumptions were made for the development of this model considering both crisp and fuzzy environments:

1. The rate of demand for inventory changes over time rather than remaining constant.
2. The model assumes that there will be no shortages in inventory, ensuring a continuous supply to meet demand.
3. The technological landscape and market conditions for smartphones remain relatively stable over the simulation period.
4. There is a delay allowed in payments without incurring penalties, reflecting realistic business practices.
5. The demand rate and lead time for smartphone inventory follow a known distribution with some inherent variability.
6. Ordering and holding costs are constant and known.
7. The lead time, or the time between placing an order and receiving it, is considered to be zero.

3.4 Mathematical Formulation

3.4.1 Parameter

- T : Total time horizon (periods)
- $D(t)$: Demand rate at time t
- $L(t)$: Lead time at time t
- $O(c)$: Ordering cost per order
- $H(c)$: Holding cost per unit per period
- Q: Order quantity (crisp)
- $\tilde{Q}$: Fuzzy order quantity

3.4.2 Crisp Model

- **Ordering Cost :** The cost incurred every time an order is placed.

$$C_O = \frac{D}{Q} \cdot O_C$$

where D is the total demand over the period, Q is the order quantity.

- **Holding Cost:** The cost of holding inventory over time.

$$C_H = \frac{Q}{2} \cdot H_C$$

- Total cost for the crisp model:

$$C(T) = C_O + C_H = \frac{D}{Q} \cdot O_C + \frac{Q}{2} \cdot H_C$$

Fuzzy Model

- **Fuzzy Ordering Cost:** The cost incurred every time an order is placed, with fuzziness in the order quantity.

$$\tilde{C}_O = \frac{D}{\tilde{Q}} \cdot O_C$$

- **Fuzzy Holding Cost:** The cost of holding inventory over time, with fuzziness in the order quantity.

$$\tilde{C}_H = \frac{\tilde{Q}}{2} \cdot H_C$$

- Total Fuzzy Cost C (T)

$$\tilde{C}(T) = \tilde{C}_O + \tilde{C}_H = \frac{D}{\tilde{Q}} \cdot O_C + \frac{\tilde{Q}}{2} \cdot H_C$$

3.5 Crisp Inventory System Model

3.5.1 Parameter

- **Periods:** Number of time periods for the simulation, here it is preset to 100.
- **Average Demand Rate:** Using global smartphone sales data from 2007 to 2023 to make the calculations
- **Demand Rate:** Introduces a 10
- **Lead Time:** Adds variability to the lead time (time between ordering and receiving inventory) by using a normal distribution with an average of 2 periods and a standard deviation of 0.5.
- **Ordering Cost:** The fixed cost incurred each time an order is placed, set at 50 units. This parameter influences the total cost in the crisp model.

- **Holding Cost:** The cost of holding one unit of inventory for one period, set at 1 unit. This cost accumulates over time as inventory is held in stock.

3.5.2 Simulation for Crisp Model

Matlab was used to create model initialization parameters such as initializing variables such as inventory level, total cost and number of orders by initializing inventory level and total cost to zero, which gives the definition of subsequent conditions. The average demand rate was evolved using global smartphone sales from 2007 to 2023 and stochasticity was added to specific parameters such as demand rate and lead time by adding mean and standard deviation, which provided stochasticity to the data and better modeled the variation of the data in the real environment. The ordering condition was then defined by checking if the current inventory level was below the required level (demand rate times lead time), if so, 100 units were ordered (crisp model EOQ), the total cost was updated by adding the cost of ordering, and the inventory level was increased by the number of orders. Next, the demand rate is used as a variable that affects the inventory level and provides real-time feedback on inventory updates based on changes in the demand rate.

Finally total cost is updated by adding holding cost (the cost of holding the remaining inventory), which simulates whether or not the cost has increased. By defining inventory levels, costs, and order quantities for the different variables under the above conditions, this in a real-world environment and is modeled on an Economic Order Quantity (EOQ) basis.

3.6 Fuzzy Inventory System Model

3.6.1 Define the Fuzzy Inference System (FIS)

Initialization of Fuzzy Inference System (FIS) is carried out to create a Mamdani-type fuzzy inference system using Matlab. This system is used to carry out the fuzzification and defuzzification of data, which is the process of converting crisp input values into the affiliation of fuzzy sets, then performs defuzzification to produce a crisp output. First the inputs and output of each variable are defined and their are categorized into different intervals across.

Define input variables with (Demand Rates) ranging from 50 to 150, with Gaussian membership functions of "Low", "Medium" and "High". (Lead time) ranges from 1 to 5 and has Gaussian membership functions for "Short", "Moderate", and "Long". Define the output variable, (Order Quantity), ranging from 0 to 200, with Gaussian membership functions for "Small", "Medium" and "Large". Next, define the rules to create a set of fuzzy rules that define the relationship between the inputs (Demand Rate and Lead Time) and the output (Order Quantity) to perform the fuzzification process. Output Membership Function is used to incorporate these rules into the fuzzy inference system. The system will use these rules during evaluation (when evalfis is called) to determine the output based on the input conditions.

Fuzzy rules are rule bases that define how to map input variables to output variables using fuzzy logic. For example, a rule might specify that the order quantity should be large if the demand rate is high and the lead time is short. and the defuzzification process is a call to the evalfis function, FIS uses the fuzzy rules and the affiliation degree from the defuzzification process to compute a fuzzy output. This fuzzy output is then defuzzified to a crisp value, in this case the order quantity.

3.6.1.1 Define Rule Base

Rule Format: Each cases where the rule format is defined as one row and five columns, representing as a fuzzy rule in the format [input1 input2 output weight operator]. Here's what each element means:

- **input1:** Index of the membership function for the first input variable (Demand Rate).

- **input2:** Index of the membership function for the second input variable (Lead Time).
- **output:** Index of the membership function for the output variable (Order Quantity).
- **weight:** The weight of the rule, indicating full importance.
- **operator:** The logical operator used to combine the inputs, use 1 for AND operation.

Example Rule: [1 1 1 1 1] : This rule means if (Demand Rate) is "Low" (index 1) and (Lead Time) is "Short" (index 1), then (Order Quantity) should be "Small" (index 1), with full weight (1) and using the AND operator (1). Breakdown of Rules:

Rule 1: [1 1 1 1 1]

If (Demand Rate) is "Low" and (Lead Time) is "Short", then (Order Quantity) is "Small".

Rule 2: [1 2 2 1 1]

If (Demand Rate) is "Low" and (Lead Time) is "Moderate", then (Order Quantity) is "Medium".

Rule 3: [1 3 3 1 1]

If (Demand Rate) is "Low" and (Lead Time) is "Long", then (Order Quantity) is "Large".

Rule 4: [2 1 1 1 1]

If (Demand Rate) is "Medium" and (Lead Time) is "Short", then (Order Quantity) is "Small".

Rule 5: [2 2 2 1 1]

If (Demand Rate) is "Medium" and (Lead Time) is "Moderate", then (Order Quantity) is "Medium".

Rule 6: [2 3 3 1 1]

If (Demand Rate) is "Medium" and (Lead Time) is "Long", then (Order Quantity) is "Large".

Rule 7: [3 1 2 1 1]

If (Demand Rate) is "High" and (Lead Time) is "Short", then (Order Quantity) is "Medium".

Rule 8: [3 2 3 1 1]

If (Demand Rate) is "High" and (Lead Time) is "Moderate", then (Order Quantity) is "Large".

Rule 9: [3 3 3 1 1]

If (Demand Rate) is "High" and (Lead Time) is "Long", then (Order Quantity) is "Large".

3.6.2 Simulation for Fuzzy Model

Similar to the crisp inventory model, after defining the fuzzy functions and rules, the definition of initialization variables such as fuzzy demand rate is carried out and the initial inventory level and total cost are set to zero. Initialize the arrays used to store the number of orders, inventory level and total cost for each period. And use these variables to track and create prerequisites for inventory levels, total costs, and order quantities. Orders are placed by evaluating the inputs (demand rate and lead time) using fuzzy logic system to determine the order quantity and if the inventory level is lower than the determined order quantity, the order is placed. Total cost is updated by adding the high end of the fuzzy ordering cost range and the impact of inventory levels will be determined based on the fuzzy order quantity.

The updated inventory data will reduce the inventory level to the high end of the fuzzy demand rate range and update the total cost by adding the high end of the fuzzy holding cost range. The above steps can be implemented to simulate an inventory system in a fuzzy environment that is based on a fuzzy environment created from input and output data after fuzzification and defuzzification of the data under a crisp inventory system. By using this fuzzy inventory system, we can well compare the inventory in both environments and whether the cost can be reduced after fuzzifying the data from the crisp environment and this study can well observe the difference between fuzzy and crisp.

3.6.2.1 Parameter

- **Fuzzy Ordering Cost:** The range of possible ordering costs for the fuzzy model, representing uncertainty in costs.
- **Fuzzy Holding Cost:** The range of possible holding costs for the fuzzy model, representing uncertainty.
- **Fuzzy Demand Rate**: The range of possible demand rates for the fuzzy model, capturing variability in demand.
- **Fuzzy Lead Time:** The range of possible lead times for the fuzzy model, capturing variability in lead times.

3.7 T Test

A t-test was conducted to determine if there was a statistically significant difference between the total costs incurred by the crisp and fuzzy inventory management systems.

- **Null Hypothesis (H0):** There is no significant difference in the total costs between the fuzzy and crisp inventory management systems.
- **Alternative Hypothesis (H1):** The fuzzy inventory management system results in significantly lower total costs compared to the crisp inventory management system.

Decision Rule:

- Inputs: crisp total costs and fuzzy total costs are arrays containing the total costs for each period under the crisp and fuzzy systems, respectively.
- **Output:** p value is the p-value from the t -test, which indicates the probability that the observed difference between the two samples is due to random chance.
- Significance Level: A p-value less than 0.05 is considered statistically significant.
- If p value < 0.05, reject the null hypothesis (H0). This indicates that there is a significant difference between the total costs of the two systems, supporting the claim that the fuzzy system results in lower costs.
- If p value >= 0.05, fail to reject the null hypothesis
- (H0). This means that the difference in total costs is not statistically significant.

3.8 Multiple Linear Regression (MLR)

Linear regression is a modeling technique that makes predictions by analyzing data. In simple linear regression, a bivariate model is built to predict the response variable (x) based on the explanatory variable (y) 1. In multiple linear regression, the model is extended to include multiple explanatory variables (x1, x2, ..., xn), thus generating a multivariate model. (Tranmer, M, 2020)

In this study, Matlab will be used to create multiple linear regression, which is a statistical technique that models the relationship between a dependent variable and multiple independent variables through the following equation.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon \qquad (1)$$

where y is total cost, x_1 is demand rate and x_2 is lead time. Where by:

y: Dependent variable (Total Cost).

x_1: Independent variable 1 (Demand Rate).

x_2: Independent variable 2 (Lead Time).

β_0: Intercept.

β_1: Coefficient for x_1.

β_2: Coefficient for x_2.

ε : Error term.

3.9 Fuzzy Logic Surface

The Fuzzy Logic Surface is a 3D surface diagram visualizing how the fuzzy logic system determines order quantities based on different demand rates and lead times. The relationship between the variables is defined as the x-axis represents the demand rate, the y -axis represents the lead time, and the z-axis represents the final order quantity determined by the fuzzy logic system.

The impact of demand rates and delivery lead times can be understood by looking at the graphs. The surface diagram shows how changes in demand rates and delivery lead times affect the number of orders, and it clearly visualizes the behavior and decision-making process of a fuzzy logic system.

Also the graphs show that the shape of the surface can reveal non-linear interactions between demand rates and delivery cycles, for example, if the surface has a steep slope or curvature, it indicates that small changes in demand rates or delivery cycles can significantly affect the number of orders. Also this graph can be used to perform sensitivity analysis by observing the sensitivity of the order quantity to changes in demand rate and delivery lead time, areas with steep slopes indicate high sensitivity, indicating that small changes in the inputs can lead to large changes in the outputs.

This fuzzy logic surface visually explains how the fuzzy logic system makes decisions based on the demand rate and delivery lead time, and it shows a continuous and smooth transition of the order quantity as the input changes, reflecting the flexibility of the fuzzy system.

3.10 Limitations

While this study provides valuable insights into the comparative analysis of inventory management systems using crisp and fuzzy environments, it is important to recognize that some limitations may affect the generalizability and robustness of the findings.

- Fuzzy Relative Function: The selection of Gaussian affiliation functions and their parameters is based on assumptions that may not be optimal for all scenarios. The effectiveness of a fuzzy model depends greatly on the appropriateness of these affiliation functions. Exploring different types of affiliation functions and fine-tuning their parameters may improve the performance of the model.
- Limited range of fuzzy variables: The fuzzy inventory management system in this study considers a limited range of fuzzy variables such as demand rate and lead time. Other factors, such as supplier reliability, transportation costs and market conditions, which have a significant impact on inventory management, were not included. The inclusion of a wider range of variables could provide a more comprehensive analysis.
- Sample size and simulation period: This study was conducted over a fixed number of periods (100), which may not be sufficient to capture long-term trends and seasonal variations in inventory systems. A longer simulation period under changing conditions could provide more reliable results and help identify potential long-term strengths or weaknesses of the fuzzy model.

IV. RESULT AND DISCUSSION

4.1 Statistical Analysis and T Test

Total cost for crisp model: 5000.00 Total cost for fuzzy model: 4000.00

Reject H0. The fuzzy inventory management system results in significantly lower costs (p-value = 0.0071).

Based on above data, we can know that the total cost under crisp inventory system is 5000 and total cost under fuzzy inventory system is 4000. The cost difference between the two types of inventory systems is 1000, and the fuzzy inventory system reduces the cost by 20% compared to the crisp inventory system, so it can be concluded that the fuzzy inventory system is superior to the crisp inventory system in terms of its ability to handle costs. The above conclusion is given to the defined fuzzy system and does not directly represent the fuzzy system in reality.

After arriving at the total cost of the inventory system in both environments, a hypothesis test is conducted and the hypothesis test method is used to determine if there is a statistically significant difference in the total cost between the crisp and fuzzy models.

- **Null Hypothesis (H0):** There is no significant difference in total costs between the crisp and fuzzy models.
- **Alternative Hypothesis (H1):** The fuzzy inventory management system results in significantly lower total costs.

In this analysis, the p-value obtained from the t-test is 0.0071. Since this p-value is less than the level of significance (0.05), we reject the null hypothesis. This indicates that the fuzzy inventory management system results in significantly lower total costs as compared to the crisp model. The p-value of 0.0071 strongly suggests that the observed difference in total cost is not due to random variation but due to increased efficiency of the fuzzy model.

4.2 Multiple Linear Regression Analysis

4.2.1 Crisp Model Analysis

In this study, the intercept is the benchmark cost estimated when both the demand rate and lead time are zero. According to Figure 7, the intercept under the explicit system is 2791 with a p-value of 0.084506, indicating that the benchmark cost associated with the explicit model is not significant. This benchmark cost does not exist independently of the demand rate and lead time and reflects the cost inherent in the inventory system. Meanwhile, the coefficient of demand rate is -0.44691. although this indicates that total cost decreases as demand rate increases, the p-value of 0.763 is higher than the 0.05 level of significance. Therefore, this negative correlation is not statistically significant. This means that fluctuations in the demand rate do not have a strong predictable effect on total costs in the explicit model.

The coefficient of lead time is 89.548 indicating a positive correlation between total cost and lead time. However, the p-value of 0.77096 is much higher than 0.05 indicating that this relationship is not statistically significant. This implies that the change in lead time does not significantly affect the total cost in the crisp inventory system.

The R-squared value of 0.00176 indicates that the model explains less than 0.2% of the variance in total cost. The adjusted R-squared is negative (-0.0188), further indicating a poor model fit.The F-statistic (0.0853) and its associated p-value (0.918) confirms that the model is not much better than the model without predictors. Therefore, the model has limited predictive power.

4.2.2 Fuzzy Model Analysis

Similar to the crisp model, in this study, here the intercept is the baseline cost estimated when both the fuzzy demand rate and the fuzzy lead time are zero. Based on Figure 8, the fuzzy model has an intercept of 2232.8 and a p-value of 0.084506, indicating that the fuzzy model don't have a significant base cost. Like the crisp model, this base cost exists independently of the demand rate and lead time, reflecting the base cost inherent in the fuzzy inventory system. The coefficient of demand rate is -0.35753 which indicates that there is a slight negative relationship between demand rate and total cost. However,

```
Multiple Linear Regression for Crisp Model:

Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate      SE        tStat       pValue
                   ________    ______    ________    ________

    (Intercept)        2791    1601.3       1.743    0.084506
    x1             -0.44691    1.4779    -0.30239       0.763
    x2               89.548    306.73     0.29194     0.77096


Number of observations: 100, Error degrees of freedom: 97
Root Mean Squared Error: 1.46e+03
R-squared: 0.00176,  Adjusted R-Squared: -0.0188
F-statistic vs. constant model: 0.0853, p-value = 0.918
```

Figure 7: Multiple Linear Regression for Crisp Model

The p-value of 0.763, which is higher than significance level 0.05, indicates that this relationship is not statistically significant. This implies that changes in the demand rate do not significantly affect the total cost in the fuzzy model.

```
Multiple Linear Regression for Fuzzy Model:

Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate      SE        tStat       pValue
                   ________    ______    ________    ________

    (Intercept)      2232.8      1281       1.743    0.084506
    x1             -0.35753    1.1823    -0.30239       0.763
    x2               71.638    245.39     0.29194     0.77096


Number of observations: 100, Error degrees of freedom: 97
Root Mean Squared Error: 1.17e+03
R-squared: 0.00176,  Adjusted R-Squared: -0.0188
F-statistic vs. constant model: 0.0853, p-value = 0.918
```

Figure 8: Multiple Linear Regression for Fuzzy Model

Meanwhile, the coefficient of lead time is 71.638 indicating that the total cost increases slightly with the increase of lead time. However, with a p-value of 0.77096, this relationship is also not statistically significant, indicating that the change in lead time does not significantly affect the total cost in the fuzzy system. The R-squared value of 0.00176 indicates that the model explains less than 0.2% of the total cost variance. The adjusted R-squared is negative (-0.0188) indicating a poor model fit. The F-statistic (0.0853) and its associated p-value (0.918) confirms that the model is not much better than the model without predictors. Therefore, the predictive power of the model remains limited.

4.2.3 Comparison of the two models

Neither model have significant intercepts, indicating that the significant baseline costs inherent in the inventory system are weakly related to demand rates and lead times, and that neither variable significantly affects total costs in the crisp or fuzzy models. This suggests that other factors not included in the regression model may have a greater impact on total costs. Also, the lower R-squared values for both models highlight the complexity of inventory management systems and indicate the need for more comprehensive models that include more variables to better predict and manage costs.

Despite the weaker predictive power of the regression model, the fuzzy inventory management system offers significant advantages over the crisp model in terms of total cost reduction, as shown by the results of the hypothesis testing and the overall cost comparison. In summary, while the MLR analysis provides some insights, it also emphasizes the need for more robust models to adequately capture the dynamics of inventory management systems. The ability of the fuzzy model to reduce costs highlights its potential to improve the efficiency of inventory management, even though the specific effects of demand rates and lead times are not highly predictive in this analysis.

4.3 Total Costs Under Different Time Periods

Figure 9 is a bar graph comparing the total costs incurred by the crisp and fuzzy inventory management systems over a 100 time period. This visualization helps to illustrate the difference in cost efficiency between the two models.

Based on Figure 9, the total cost of the crisp model (blue bars) increases steadily over time, with a slightly steeper upward trend as the trend changes over time. This indicates a linear pattern of cost growth with some variability, reflecting how the crisp inventory system responds to changing demand and lead time conditions. The total costs (red bars) of the fuzzy model also increase over time, but at a slower and more consistent rate than the crisp model. The fuzzy model is less volatile, indicating a more stable and predictable cost model. By comparing and analysing the two inventory systems, the total cost of both models is relatively low in the initial stage. The cost of the fuzzy model is slightly lower than that of the crisp model, indicating higher efficiency in the early stages, but the gap between the total costs of the crisp and fuzzy models grows wider over time. By the end of 100 cycles, the total cost of the crisp model is significantly higher than the fuzzy model. This indicates that the fuzzy inventory management system has the potential for long-term cost savings. Therefore, it can be said that the fuzzy model consistently reduces the total cost compared to the crisp model. This overall efficiency improvement highlights the effectiveness of the fuzzy system in managing inventory more economically in the long run. Meanwhile, the crisp model exhibits fluctuations in total cost, reflecting its sensitivity to changes in demand and lead times. These fluctuations can be attributed to the rigidity of traditional inventory systems, which do not adapt well to changing conditions.

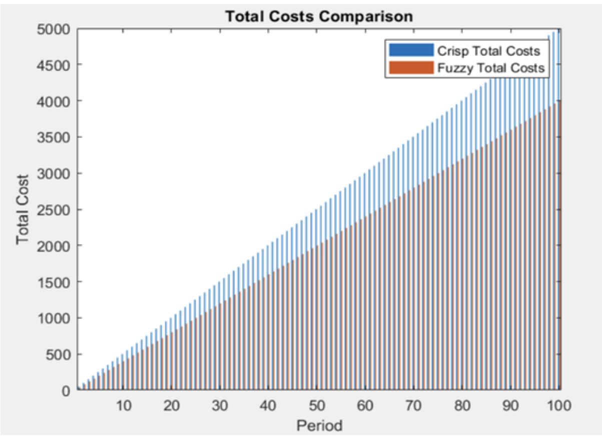

Figure 9: Total Costs Comparison

The fuzzy inventory management system has a low long-term total cost of ownership, which underscores its superior efficiency. By using fuzzy logic, the system can better handle uncertainties and variations in demand and lead times, resulting in more optimized order quantities and inventory levels. At the same time its predictability and more stable cost patterns indicate better control over inventory costs. This predictability is critical to the business as it enables more accurate budgeting and financial planning. Businesses that adopt fuzzy inventory management can expect significant cost reductions as the system continues to operate.

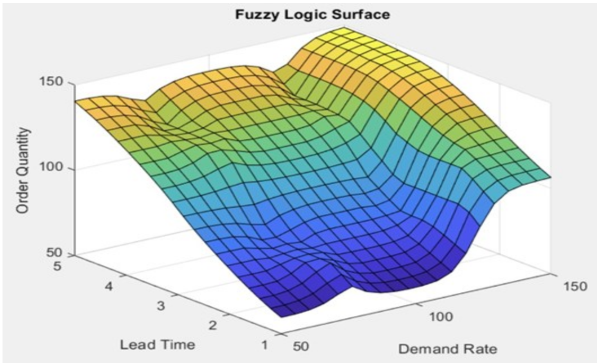

Figure 10: Fuzzy Logic Surface

4.4 Fuzzy Logic Surface

Figure 9 clearly illustrates the cost efficiency of the fuzzy inventory management system compared to the traditional crisp model. The stability and lower total cost of the fuzzy model confirms its effectiveness in reducing inventory costs. This visualization combined with statistical analysis provides strong evidence of the benefits of using fuzzy logic in inventory management.

Figure 10 is a 3D surface diagram illustrating the relationship between demand rates, lead times and order quantities as determined by the fuzzy logic system. The figure visualizes how the fuzzy inventory management system adjusts the order quantity based on different demand rates and lead times.

By looking at the surfaces, smooth transitions and varying slopes are shown, indicating that the fuzzy system is able to handle different scenarios flexibly. The gradient of the surface changes smoothly, which is typical for fuzzy logic systems that rely on asymptotic membership functions rather than binary decisions. As well as observing the demand rate it is found that as the demand rate increases (moving from left to right along the x -axis) the number of orders typically increases. This is intuitive because higher demand rates require larger order quantities to maintain inventory levels and meet customer demand. For lower demand rates, order quantities are relatively small, reflecting adjustments made by the system to prevent excess inventory and minimize holding costs.

At the same time, as lead time increases (moving from front to back along the y-axis), order quantity initially increases, but then exhibits more complex behavior. Short lead times result in lower order volumes because replenishment can happen quickly. Moderate lead times require higher order quantities to ensure sufficient inventory is available during the waiting period. For very long lead times, order volumes may again decrease as the system balances the risk of excess inventory with the need for extended waiting periods.

The surface diagram of a fuzzy logic system demonstrates its ability to make adaptive decisions based on changing conditions. Unlike traditional models that may apply the same rules uniformly, fuzzy systems dynamically adjust their recommendations (order quantities). The smooth and continuous surface reflects the ability of the fuzzy system to handle uncertainty and variability in the input variables. This is critical in real-world inventory management, where demand rates and lead times may fluctuate unpredictably.

Figure 10 clearly shows how the system responds to varying demand rates and lead times, optimizing order quantities to maintain cost efficiency and inventory balance. This adaptive capability highlights the advantages of using fuzzy logic in inventory management and is consistent with the statistical results showing that fuzzy systems excel in reducing total cost.

V. CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In conclusion, this study aims to compare the efficiency of crisp and fuzzy inventory systems in terms of total cost.

Through detailed simulations, statistical analysis and visualization, the following conclusions were drawn.

Fuzzy models offer significant advantages in reducing the total cost of ownership compared to traditional crisp systems. Fuzzy systems can effectively deal with uncertainties in demand rates and lead times, leading to more optimized decisions. Fuzzy systems maintain lower and more stable costs as the period increases, so companies can benefit from adopting a fuzzy inventory management system, resulting in significant cost reductions. And the robustness of fuzzy models in dealing with variability and uncertainty makes them a valuable tool for managing the risks associated with supply chain disruptions and demand fluctuations, which can lead to more resilient and responsive inventory management practices.

The comparative analysis of crisp and fuzzy inventory systems provides compelling evidence of the superiority of the fuzzy approach, and the results of the study emphasize the importance of utilizing advanced methods such as fuzzy logic to enhance inventory management practices. Overall, the adoption of fuzzy inventory systems is highly beneficial for companies seeking to optimize their inventory processes and achieve sustainable cost savings.

5.2 Recommendations

My recommendations is to combine advanced technologies such as artificial intelligence and machine learning with fuzzy logic to further enhance inventory management. These technologies can provide real-time data analytics, predictive insights, and more granular decision-making capabilities. Companies should also conduct further research to explore the wider application of fuzzy logic in all areas of inventory management. This includes examining its impact on different types of inventory, industries and supply chain configurations. In conclusion, inventory management is a dynamic field that requires continuous improvement and adaptation. Organizations should stay abreast of the latest advances in inventory management techniques, and adopting a holistic approach that combines fuzzy logic with other state-of-the-art methods and techniques can significantly improve inventory management. By following these recommendations, organizations can achieve more efficient and effective inventory management, resulting in lower costs, higher service levels and improved overall performance.

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